6.17 The times spent studying by students in the week before final exams follows a normal distribution with standard deviation 8 hours. A random sample of four students was taken in order to estimate the mean study time for the population of all students.

a. What is the probability that the sample mean exceeds the population mean by more than 2 hours?
b. What is the probability that the sample mean is more than 3 hours below the population mean?
c. What is the probability that the sample mean differs from the population mean by more than 4 hours?
d. Suppose that a second (independent) random sample of 10 students was taken. Without doing the calculations, state whether the probabilities in parts (a), (b), and (c) would be higher, lower, or the same for the second sample.

a. $\sigma_{\bar{x}} = \frac{8}{\sqrt{4}}$ = 4 hours. $P(Z > 2/4) = P(Z > .5) = .3085$

b. $P(Z < -3/4) = P(Z < -.75) = .2266$

c. $P(-4/4 > Z > 4/4) = P(-1 > Z > 1) = .3174$

d. Lower, lower, lower

6.30 In 1992, Canadians voted in a referendum on a new constitution. In the province of Quebec, 42.4% of those who voted were in favor of the new constitution. A random sample of 100 voters from the province was taken.

a. What is the mean of the distribution of the sample proportion in favor of a new constitution?
b. What is the variance of the sample proportion?
c. What is the standard error of the sample proportion?
d. What is the probability that the sample proportion is more than 0.5?

a. $E(\hat{p}) = .424$

b. $\sigma_{\hat{p}}^2 = \frac{(.424)(.576)}{100} = .00244$

c. $\sigma_{\hat{p}} = .0494$

d. $P(Z > \frac{.5 - .424}{.0494}) = P(Z > 1.54) = .0618$

6.54 In a large city it was found that summer electricity bills for single-family homes followed a normal distribution with a standard deviation of $100. A random sample of 25 bills was taken.

a. Find the probability that the sample standard deviation is less than $75.
b. Find the probability that the sample standard deviation is more than $150.

a. $P(\frac{(n-1)s^2}{\sigma^2} < \frac{24(75)^2}{(100)^2}) = P(\chi^2_{(24)} < 13.5)$ = between .025 and .05 (.0429 exactly)

b. $P(\frac{(n-1)s^2}{\sigma^2} > \frac{24(150)^2}{(100)^2}) = P(\chi^2_{(24)} > 54)$ = less than .005 (.0004 exactly)